

Chiral Symmetry from Type IIA Branes

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We propose a mechanism in which, using an eightbrane, a sixbrane ends on a NS brane in type IIA superstring theory. We use this mechanism to construct $N = 1$ supersymmetric gauge theories in four dimensions with chiral matter localized in different points in space. Anomaly cancellation for the gauge theories is satisfied by requiring RR charge conservation for the various type IIA fields. The construction allows us to study a curious phase transition in which the number of flavors in supersymmetric QCD depends on the value of the ten dimensional cosmological constant. These phenomena are related to the fact that every time a D8-brane crosses a NS brane, a D6-brane is created in between them.

1. Introduction

By now it is clear that D-branes [1] are a very useful tool in constructing gauge theories in various dimensions and with various supersymmetries. Such a construction has proved to be a very useful tool in studying the dynamics of the gauge theories constructed. In [2] it was proposed how to construct a brane configuration which describes gauge theories with 8 supercharges in three dimensions. This construction can be T-dualized and give similar configurations in various dimensions as was exploited in [3,4,5,6]. Application of the methods in [2] were performed also in [7]. In addition see some recent generalization of the work of [3] in [8,9].

The configuration in [2] was generalized to construct gauge theories with 4 supercharges in four dimensions in [10]. Applications of this approach can be found in [11,12,13,14,15,16]. A different approach to study the same class of theories, consisting in wrapping D-branes on Calabi-Yau cycles, is analyzed in [17,18,19,20,21,22,23].

The theories constructed so far with 4 supercharges are vector like. It is very natural to try and construct chiral gauge theories and study their dynamics using the new brane methods. It should be mentioned that chiral matter from branes was constructed in [24] and the anomaly was canceled by space-time fields. This will not be the approach we will take in this paper.

A first step toward understanding the construction of chiral gauge theories was done in [13] where the chiral symmetry of supersymmetric QCD was identified in terms of the branes. When sixbranes parallel to NS branes meet in space, the sixbranes can break and form two different gauge groups. In [13] it was also proposed how chiral multiplets arise when three branes meet at the same point in space time. Concretely, the proposal was that when a semi-infinite D6 brane meets a D4 brane on the world-volume of a NS fivebrane, there is a massless chiral multiplet in the spectrum. Such a brane system is far from being described by the usual D-brane methods and thus, unfortunately it is not possible to calculate directly how such a multiplet arises. However we can look for some consistency checks which will give support for this proposal.

In [4] a rather nontrivial check on the proposal in [13] was performed. The superpotentials of various brane configurations were calculated. It was found that precisely at the points where the superpotential is zero - where chiral symmetry is expected - sixbranes can actually meet parallel NS branes and then break.

The approach we will take in this paper is to assume that this proposal is correct and study its consequences. In doing this we find some more consistency conditions which

supports this proposal. We find new interesting phenomena which arise due to this assumption. This also provides the next step towards constructing a wide class of theories with chiral matter.

A first question which arises when a semi-infinite sixbrane ends on a NS fivebrane in type IIA superstring theory is that there is lack of gauge invariance for the RR field which carries the D6-brane charge. In other words, the charge of the D6-brane can not flow anywhere when the D6-brane ends on a NS brane. We will first address this question and solve it by introducing D8-branes. Such branes do not break further the supersymmetry as will be discussed in section 2.1 . These D8-branes can be considered to be located far away from the system of D4-branes to avoid new massless states in the four dimensional theory. Their effect is to create a cosmological constant for the type IIA string theory. We will show that for non-zero value of the cosmological constant a semi-infinite sixbrane can end on a NS fivebrane. In this setup, if a D4-brane ends on the NS-brane and meets the D6-halfbrane, there is a massless chiral multiplet in the spectrum.

However a single chiral multiplet is anomalous in four dimensions and thus we expect some inconsistencies to arise. Indeed this is what happens and whenever we try to construct gauge theories with anomalous matter there is some RR-charge violation in the string theory setup. Conversely, only configurations with no charge violations give rise to anomaly free theories in four dimensions. Starting with configurations which obey the RR-charge conservation the theory may undergo a phase transition which naturally preserve the RR-charge. Thus the theories beyond the phase transition remain anomaly free. Chiral multiplets can however be supported near one of the boundary of the D4-brane. The KK reduced four dimensional theory is anomaly free, but one may have local anomalies supported on the boundaries of the segment on which the theory is defined. All anomalies, global or localized on the boundaries, must cancel in a consistent theory, and we indeed show how the presence of a cosmological constant induces Chern-Simons terms on the D4-worldvolume which precisely cancel the anomalies on the boundaries.

The cancellation of the RR charge in spacetime, for the configurations considered in this paper, corresponds to a non-chiral four dimensional fields content. If we move background branes around in space, the four dimensional theory may undergo phase transitions. We consider here only final configurations which are still non-chiral. The matter is however localized in different points in space according to its chirality. In one of these phase transitions, we connect $N = 1$ gauge theories with different number of flavors. This phenomenon is obtained by changing a real parameter in spacetime (the position of a D8-brane) and has

no field theoretical counterpart in the four dimensional theory, where a complex parameter is required to give mass to the quarks. The number of flavors is connected to the value of the ten dimensional cosmological constant, which is indeed quantized.

The cancellation of the RR charge gives informations also about the six-dimensional gauge theory living on the world-volume of the D6-branes. We show in section 3, that the charge conservation in spacetime exactly reproduces the anomaly cancellation condition for the six-dimensional gauge theory. In the meanwhile, we point out that, whenever a D8-branes crosses a NS-brane, a D6-brane is created in between them. This phenomenon is T-dual to the one originally discussed in [2], where a D3-brane is created in between D5 and NS-branes. It is also U-dual to the creation of a fundamental string between D0 and D8-branes recently considered in [25] and other examples of branes creation in [26].

The paper is organized in the following way. In section 2, we first review the proposal for chiral symmetry in [13] and extend it to SO and USp gauge groups. We discuss then how a D6-brane can be finite without violating the RR charge conservation. The D6-brane must end on a D8-brane or on a NS-brane in the presence of non-zero cosmological constant. In section 3, the system consisting of D6, D8 and NS-branes is applied to the study of six-dimensional $N = 1$ gauge theories. The phenomenon for which a D6-brane is created every time a D8-brane crosses a NS-brane is crucial for the correct understanding of the six-dimensional theory. In section 4, we apply the set-up of section 2 and 3 to four-dimensional $N = 1$ gauge theories, showing how to localize the chiral matter in different points of the space. A curious phase transition in which the number of flavors depends on the value of the cosmological constant is also presented. Section 5 contains some brief concluding remarks. The appendix contains the normalizations for the RR charges and for the parameters in the massive type IIA Lagrangian.

2. Half D6-branes, massive type IIA and chirality

2.1. A dictionary for the branes configuration

In this section we introduce the configurations and present the setup of the construction. We recall the dictionary of how states arise in various configurations. Next we recall the problem of how to identify the global chiral symmetry in $N = 1$ gauge theory models built with configuration of branes in type IIA string theory. The set-up is the same as in [10], where it was introduced to explain Seiberg's duality, adapting a construction first appeared in [2].

The ingredients of the construction are Dirichlet (D) fourbranes, Dirichlet sixbranes and Neveu-Schwarz (NS) fivebranes in type IIA string theory. They share the space-time directions (x^0, x^1, x^2, x^3) where the four-dimensional $N = 1$ gauge theory is realized. The D6-branes and the NS-branes are infinite in the remaining directions and can be considered as infinitely heavy background branes. The D4-branes are stretched between D6 or NS-branes in the x^6 direction. Thus they are finite in the x^6 direction and their world-volume theory is on an interval. After Kaluza-Klein reduction, the D4-branes theory is a four-dimensional gauge theory in which the world-volume fields of the D6 or NS-branes appears as background fields associated with global symmetries.

More explicitly, the ingredients are:

- (1) NS fivebrane with world-volume $(x^0, x^1, x^2, x^3, x^4, x^5)$, which lives at a point in the (x^6, x^7, x^8, x^9) directions. The NS fivebrane preserves supercharges of the form¹ $\epsilon_L Q_L + \epsilon_R Q_R$, with

$$\begin{aligned}\epsilon_L &= \Gamma^0 \cdots \Gamma^5 \epsilon_L \\ \epsilon_R &= \Gamma^0 \cdots \Gamma^5 \epsilon_R.\end{aligned}\tag{2.1}$$

- (2) NS' fivebrane with world-volume $(x^0, x^1, x^2, x^3, x^8, x^9)$, which lives at a point in the (x^4, x^5, x^6, x^7) directions, preserving the supercharges

$$\begin{aligned}\epsilon_L &= \Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3 \Gamma^8 \Gamma^9 \epsilon_L \\ \epsilon_R &= \Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3 \Gamma^8 \Gamma^9 \epsilon_R.\end{aligned}\tag{2.2}$$

- (3) D4-brane with world-volume $(x^0, x^1, x^2, x^3, x^6)$, which lives at a point in the $(x^4, x^5, x^7, x^8, x^9)$ directions and preserves supercharges satisfying

$$\epsilon_L = \Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3 \Gamma^6 \epsilon_R.\tag{2.3}$$

- (4) D6-brane with world-volume $(x^0, x^1, x^2, x^3, x^7, x^8, x^9)$, which lives at a point in the (x^4, x^5, x^6) directions. The D6-brane preserves supercharges satisfying

$$\epsilon_L = \Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3 \Gamma^7 \Gamma^8 \Gamma^9 \epsilon_R.\tag{2.4}$$

We will need in the following also $D6'$ -branes spanning $(x^0, x^1, x^2, x^3, x^4, x^5, x^7)$ and D8 brane with world volume $(x^0, x^1, x^2, x^3, x^4, x^5, x^6, x^8, x^9)$.

¹ Q_L, Q_R are the left and right moving supercharges of type IIA string theory in ten dimensions. They are (anti-) chiral: $\epsilon_R = -\Gamma^0 \cdots \Gamma^9 \epsilon_R$, $\epsilon_L = \Gamma^0 \cdots \Gamma^9 \epsilon_L$.

Each brane by itself would break $\frac{1}{2}$ of the supersymmetry, but it is easily checked that the relations (2.1)-(2.4) are not independent. It was found in [2] for a T-dual configuration that combining the relations (2.1) and (2.3) leads to the relation (2.4). Thus in the presence of the D4-branes and the NS-brane, the addition of the D6-brane does not break further the supersymmetry and there are 8 unbroken supersymmetries. Once the NS' brane is introduced, the supersymmetry is reduced further to 4 unbroken supercharges. Now it is possible to combine all possible supersymmetry conditions and look for new types of branes for which the presence will not break supersymmetry further [27]. For example combining equation (2.2) and equation (2.3) leads to a supersymmetry condition which is obeyed in the presence of D6'-branes. A combination of equation (2.2) and equation (2.4) leads to a supersymmetry condition which is obeyed in the presence of a D8-brane. It is easy to check that combination of all other supersymmetry conditions does not lead to new types of branes other than the ones mentioned above [27]. There is of-course a possibility to rotate the NS branes and the D6 branes by an arbitrary angle in the 45 - 89 directions as discussed in [4] and applied in [13]. We will not use this in the present paper but it would be interesting to study the effects of such rotations on the low energy field theory dynamics. As a result of all these considerations, a $N = 1$ supersymmetry is realized on the D4 world-volume.

The finite D4-branes can end on either 5-branes or 6-branes. In principle, the world volume of the D4 brane contains a $N = 4$ $U(1)$ multiplet, but some massless states are projected out by boundary conditions. These conditions are different whether the D4-branes end on 5-branes or 6-branes. Let us review, [2], what fields survive in the different cases.

1) For a 4-brane between two NS 5-branes, there is a $N = 1$ vector multiplet and chiral multiplet in the adjoint representation living on the world volume theory of the 4-brane (the adjoint chiral multiplet corresponds to motion of the 4-brane in the (x^4, x^5) direction). For n coincident D4-branes, we get a $SU(n)$ gauge theory².

2) For a D4-brane between an NS 5-brane and an NS' 5-brane there is only a vector multiplet. In general, the two 5-branes can be at an arbitrary angle θ , [24], in the directions (x^4, x^5, x^8, x^9) without breaking any further supersymmetry. See also a related discussion in [28]. The mass of the adjoint chiral multiplet that appeared in item one is proportional,

² As in [3], the $U(1)$ multiplet is frozen by requiring finite energy configuration on the 5-branes.

for small angles, to the angle between the two 5-branes. For n such D4-branes we get $SU(n)$ gauge theory.

3) For a D4-brane between two D6-branes, only scalars fields survive. These scalars are associated with motion of the D4-brane along the (x^7, x^8, x^9) coordinates together with the A_6 component of the gauge field. The A_6 component is compact with a radius proportional to the gauge coupling. These four scalars are conveniently written in terms of two chiral multiplets $x_8 + ix_9$; $x_7 + iA_6$.

4) For a D4-brane between a D6-brane and an NS 5-brane there are no massless moduli which contribute to the low energy field theory. In general, this is true for any NS5-brane and D6-brane that are not parallel. For more than one D4-branes between a D6-brane and a NS brane - an S-configuration - the system is conjectured to break supersymmetry [2].

5) For a 4-brane between a D6-brane and NS' 5-brane there is one chiral multiplet $x_8 + ix_9$, which corresponds to motion of the D4 brane in between the two branes. In general there will be one chiral multiplet which is associated with the motion of a D4-brane between parallel D6-brane and a NS brane.

In addition to the above massless states, there are other fields which come from open strings stretching between the 4-branes and the 6-branes. These strings give fields in the fundamental representation of the gauge group, Q and \tilde{Q} , the quarks. They carry also indices in the fundamental of the gauge group living on the world-volume of the 6-branes. Since the 6-branes are infinite in the (x^7, x^8, x^9) directions, this gauge group is frozen from the four-dimensional point of view and appears as a global symmetry.

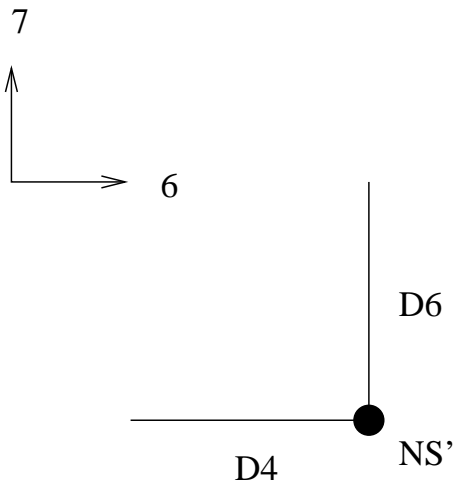


Figure 1: A system of semi-infinite D6 branes which end on a NS' brane and finite D4 branes which end on the NS'. A chiral multiplet is conjectured to arise from strings stretched between the two D branes.

Following [13], we can now add one more case for which there are massless chiral moduli. For completeness of the dictionary we state this case here however one should bare in mind that the possible charge violation and anomaly should be avoided as discussed in detail in the rest of this section.

6) When a D4 brane ends on a NS' brane which is a boundary of a semi-infinite sixbrane there is a massless chiral multiplet in four dimensions. This multiplet arises due to strings stretched between the D6 and the D4 branes. Only one orientation of the strings contributes to the massless states which leads to only one chiral multiplet, the other orientation being projected out. In the usual case in which the D6 branes are infinite both orientations of the strings contribute massless states and thus we have two chiral multiplets. When N_c branes end to the left (in the x^6 direction) of the NS' brane and N_f D6 branes end on top (in the x^7 direction), as in figure 1, the chiral multiplet transforms in the (N_c, \bar{N}_f) representation. Other cases, like when the N_c branes end to the right, transform in an obvious way keeping in mind that the orientation of the strings which contribute to the massless states remains the same.

2.2. Chiral symmetry

We now turn to some applications of the dictionary above. The simplest example, SQCD, can be easily constructed by stretching N_c D4-branes between a NS 5-brane and a NS' 5-brane, with N_f D6-branes in between the two NS-branes. The distance in the (x^4, x^5) plane between the D6 and the D4 branes is the complex mass parameter for the quarks Q, \tilde{Q} . When all the masses are zero, the D6-branes meet the D4 branes. Tuning further the x^6 positions of the D6-branes they can coincide in spacetime and form an enhanced gauge symmetry. The $SU(N_f)$ gauge theory on the D6 world-volume becomes a $SU(N_f)$ global symmetry rotating the quarks. A transition to the Higgs branch is now possible in field theory. In the brane language, it corresponds to splitting the D4-branes between the D6-branes which are now at the same point. A consistent picture of the resulting moduli space is obtained if one takes in consideration the s-rule, discovered in [2]:

A configuration in which a NS-brane and a D6-brane are connected by more than one D4-brane is not supersymmetric.

SQCD has actually an $SU(N_f)_L \times SU(N_f)_R$ chiral global symmetry, whose diagonal $SU(N_f)$ we identified with the D6 world-volume gauge theory. The remaining part of the chiral symmetry is apparently lacking in the brane construction. This is related to the

fact that most pieces in this construction are *locally* $N=2$, at least for what regards the field content. The quarks Q and \tilde{Q} , for example, are generated by the local interaction between a D4 and D6 brane system which has $N = 2$ supersymmetry in four dimension and (regarding the field content) hardly knows about the existence of distant NS branes that further break the supersymmetry to $N=1$. In the $N = 2$ supersymmetric QCD case, there is indeed a term in the superpotential,

$$W = \lambda Q X \tilde{Q}. \quad (2.5)$$

where X is the adjoint chiral superfield partner of the gauge fields under $N = 2$ supersymmetry. λ is fixed to $\sqrt{2}$ by supersymmetry. This term implies that when we give a vacuum expectation value to the adjoint field, the hypermultiplets receive a mass. This term also breaks the global chiral symmetry from $SU(N_f)_R \times SU(N_f)_L$ to $SU(N_f)$. Turning off the coefficient λ in (2.5), breaks the $N = 2$ supersymmetry to $N = 1$ while restoring the chiral symmetry.

A mechanism for identifying the missing chiral symmetry was proposed in [13]: the chiral symmetry is restored when the 6-branes touch a parallel 5-brane. To make, say a NS' brane and a D6 brane coincide, we need to tune the three coordinates (x^4, x^5, x^6) . In [13] it was proposed that a massless multiplet comes down in the world volume $(x^0, x^1, x^2, x^3, x^8, x^9)$ which is mutual to both branes. The supersymmetry in the NS'-D6 system is $(0, 1)$ in six dimensions, and the only supermultiplet which can become massless by tuning only three parameters is a vector multiplet. The distance in the (x^4, x^5, x^6) directions gets the interpretation of a FI parameter for this $U(1)$ gauge field.

The only direction that the 6-branes and the 5-brane do not share is x^7 . Therefore, a 6-brane can have a boundary on the 5-brane in the x^7 direction. Thus the proposal is that when N_f 6-branes touch a parallel 5-brane, they split in half, and the $(x^0, x^1, x^2, x^3, x^8, x^9)$ world volume theory becomes $SU(N_f)_R \times SU(N_f)_L$. Strings stretching from the N_c 4-branes to the N_f 6-branes to the right in x^7 , are the quarks Q charged under $(\mathbf{N}_c, \mathbf{N}_f, \mathbf{1})$. Strings stretching from the N_c 4-branes to the N_f 6-branes to the left in x^7 , are the quarks \tilde{Q} charged under $(\bar{\mathbf{N}}_c, \mathbf{1}, \mathbf{N}_f)$. Moving the 6-branes off the 5-brane in the (x^4, x^5) direction breaks the six dimensional symmetry spontaneously while breaking the four dimensional chiral symmetry explicitly, as it should since motion in the (x^4, x^5) corresponds in the field theory to giving the quarks a mass. Breaking the 4-branes on the 6-branes is only possible if the 6-branes move off the 5-brane in the (x^8, x^9) directions. This corresponds to

Higgsing, and in agreement with field theory, Higgsing breaks chiral symmetry. Actually, from the point of view of the six brane observer, the motion of the D6 branes in the 456 directions correspond to moving on a Higgs branch while the point when the D6 branes touch the NS' brane is the origin of this Higgs branch. There is no Coulomb branch for vector multiplets in six dimensions and thus there is no other motion from this point, associated with breaking.

The chiral symmetry proposal was further checked and explored in [4]. There the setup was mostly in the context of three dimensional $N = 2$ gauge theories realized in terms of branes which is a T-dual version of the configurations we discuss here. In [4] superpotentials were calculated for a class of brane configurations. It was found that in cases where the superpotential vanishes, that is when chiral symmetry is expected, the transition of the type described in [13] and above are possible. The calculation in [4] thus serves as a non-trivial consistency check for the proposal of [13]. One of the purposes of this paper is to provide further consistency checks for this proposal.

2.3. $SO(n)$ and $Sp(n)$ gauge groups

In this section we want to discuss the changes in the previous construction when applied to $SO(2N_c)$ and $USp(2N_c)$ gauge groups with N_f flavors. The field theory analysis predicts a maximal global symmetry $SU(2N_f)$. Only a subgroup is visible in the standard construction with infinite D6-branes. We show that, using semi-infinite D6-branes, the maximal global symmetry becomes manifest.³

$SO(n)$ and $Sp(n)$ gauge theories can be realized by introducing suitable orientifold planes [12][15]. We will consider in this section the simplest case in which the orientifold plane is parallel to the D4-branes and we will call it O4. It does not break any further supersymmetry but it creates images for all the branes in the directions x^4, x^5, x^7, x^8, x^9 transverse to the D4-plane. The space-time operation of reflecting the coordinates x^4, x^5, x^7, x^8, x^9 is accompanied by the world-sheet parity Ω . If we project out the Chan-Paton factors of $2n$ branes (n physical branes plus n images) with a symmetric unitary matrix we obtain an $SO(2n)$ gauge group for the D4-branes, while if we

³ A.H. would like to thank discussions on related issues with Per Kraus and Jaemo Park.

project with an antisymmetric unitary matrix we obtain the $USp(2n)$ gauge group⁴. The symmetric case allows to consider also an odd number $2n + 1$ of D4-branes, realizing an $SO(2n + 1)$ gauge theory. In this case, one of the D4-branes has no image and it is stucked at the orientifold point.

The D6-branes, which meet orthogonally the D4-branes, are projected by a matrix which is antisymmetric if the one which projects the D4-branes is symmetric, and vice versa [29]. In this way, the global symmetry manifest in the brane construction for $SO(2N_c)$ gauge theories is $USp(2N_f)$ and for $USp(2N_c)$ gauge theories is $SO(2N_f)$. This result is consistent with the field theoretical expectations for an $N = 2$ gauge theory. When the gauge theory is $N = 1$, from the field theory point of view, we expect a maximal $SU(2N_f)$ global symmetry. The $N = 1$ case is realized using two different orientations of solitonic five-branes, the NS and the NS'. As discussed in subsection 1, we expect to see the maximal global symmetry when the D6-branes touch the NS' and split.

We will consider only the case of an $SO(2N_c)$ gauge theory, for simplicity. The $SO(2N_c)$ gauge group is unbroken if all the $2N_c$ D4-branes are at the orientifold point, which we take at $x^4, x^5, x^7, x^8, x^9 = 0$. All the hypermultiplets are massless when the D6-branes touch the D4-branes along the orientifold plane. In a generic point in the x^6 direction, the global symmetry is $USp(2N_f)$. In general, the Chan-Paton factor for $2N_f$ branes is a generic hermitian matrix in $U(2N_f)$. The Ω projection reduces it to a matrix in $USp(2N_f)$.

However, when the D6-branes are on top of the NS'-brane, they can split, giving a total of $4N_f$ half D6-branes. The orientifold projection acts now in a totally different way. The world-volume field living on a D6-halfbrane which fills the positive x^7 direction is identified with the world-volume field on a D6-halfbrane which fills the negative x^7 direction, but no projection is needed on the $2N_f \times 2N_f$ hermitian matrix which describes the Chan-Paton factors of the half D6-branes. In this way, we find a manifest $SU(2N_f)$ global symmetry, as predicted by the field theoretical analysis.

⁴ In a superstring compactification, only the symmetric projection is allowed. In fact, the antisymmetric projection corresponds to the unusual case in which branes and orientifold planes charges have the same sign and, therefore, when the space transverse to the branes is compact there is no way to cancel the charge or, equivalently, the tadpoles. In our case, in which the transverse space is not compact, both projections are allowed.

2.4. RR Charge Conservation

It would be natural now to conjecture that a configuration in which only half of the D6-brane exists, say the semi-infinite D6-brane to the right in x^7 , would correspond to a chiral model, with only the quark Q . Unfortunately, this configuration violates the RR 7-form charge conservation. The RR charge flux of a Dp-brane ending on a (NS or RR) q-brane is usually absorbed by some fields on the q-brane world-volume. This works only when $q > p$. In our case there is no way in which the NS 5-brane can absorb the flux of the D6-brane.

This violation of charge conservation is actually a good sign from the point of view of the four dimensional field theory. A gauge theory with only one charged field Q is anomalous and thus inconsistent. This means that a violation of the RR 7-form charge is translated to an anomaly for the four dimensional gauge theory. Thus when looking for anomaly free theories we will look for string theory brane configurations which do not violate charge conservation as they must in order to be consistent⁵.

2.5. D8-branes and massive type IIA supergravity

In this section we present a mechanism for compensating the RR 7-form charge violation induced by a semi-infinite D6-brane. Let us start by writing explicitly this charge violation.

The D6-brane is charged under the type IIA 7-form $A^{(7)}$, the magnetic dual of the type IIA $U(1)$ form $A^{(1)}$. The Bianchi identity for $A^{(1)}$ in the presence of a semi-infinite D6-brane, say, in the positive x^7 direction, reads:

$$dF^{(2)} = d * F^{(8)} = \theta(x^7)\delta^{(456)} \quad (2.6)$$

Taking the differential of this equation, we derive the inconsistent relation $0 = \delta^{(4567)}$ which reveals that the RR charge is not conserved in such a configuration.

The simplest way to compensate the RR charge violation is to make the D6-brane end on a D8-brane. As we saw in §2.1 the supersymmetry still allows the existence of a D8-brane which is a point in the x^7 direction. The RR charge flux is then absorbed by

⁵ There exist also systems of branes which realize anomalous field theories on the common intersection [30]. The anomaly is cancelled by an in-flow of charge from the background branes. We will not consider this situation in this paper.

the $U(1)$ gauge field which lives on the D8 world-volume and which sees the end of the D6-brane as a magnetically charged 5-brane which we will call a magnetic monopole.

The existence of a D8-brane at $x^7 = 0$ would modify the right hand side of (2.6) in the following way

$$dF^{(2)} = d * F^{(8)} = \theta(x^7)\delta^{(456)} - F_{D8}^{(2)}\delta(x^7) \quad (2.7)$$

where $F_{D8}^{(2)}$ is the $U(1)$ gauge field strength which lives on the D8 world-volume. Differentiating (2.7) we now get the statement that the D6-brane endpoint is a magnetically charged monopole for the D8 gauge field: $dF_{D8}^{(2)} = \delta^{(456)}$.

The modification to (2.6) follows from the couplings to the RR background fields existing on the world-volume of a Dp-brane [31],

$$\int dx^{p+1} C \wedge \text{tre}^{(F-B)} \quad (2.8)$$

where C is the formal sum of all the RR background forms, F is the world-volume gauge field, B is the background NS-NS antisymmetric tensor and it is to be understood that the world-volume integral selects the $(p+1)$ -form in the expansion. In the case of a D8-brane the relevant coupling which affects the equation of motion for $A^{(7)}$ (the Bianchi identity for $A^{(1)}$) is $A^{(7)} \wedge F_{D8}^{(2)}$.

As discussed in section 2.1, four dimensional chiral matter arises when the D6-halfbrane boundary coincide with the NS'-brane on which the D4-brane ends. If a D8-brane is placed at the same point in x^7 to make the half D6-brane consistent, there is a new multiplet in the D4-worldvolume associated with the 4-8 strings. To avoid the creation of new multiplets in the four-dimensional spectrum, it would be useful to have a different mechanism for reabsorbing the RR flux of the D6-brane.

The only other way to modify (2.6) without involving new extra branes would be to get a contribution to this equation directly from the bulk type IIA supergravity Lagrangian. An easy check reveals that there are no terms in type IIA supergravity which can modify (2.6) in a way which compensates the RR charge violation. At this point it would seem hopeless to construct chiral theories as described in the last section. However a term which can compensate the charge conservation exists in the massive type IIA supergravity [32] in the form of the coupling

$$-m \int dx^{10} B \wedge *F, \quad (2.9)$$

where m is the mass parameter for the type IIA, related to the ten-dimensional cosmological constant. In a massive type IIA background, (2.6) now would read:

$$dF^{(2)} = d * F^{(8)} = \theta(x^7)\delta^{(456)} - mH. \quad (2.10)$$

Differentiating this equation, we get $mdH = \delta^{(4567)}$. We conclude that the RR charge flux of a half D6-brane can be reabsorbed in the massive type IIA background by a NS 5-brane coinciding with the boundary of the half D6-brane. This is exactly the configuration of branes which was conjectured in [13] to make manifest the chiral global symmetry of SQCD, as discussed at the end of the previous section.

Let us analyze this proposal further, showing how the massive type IIA arises naturally in the brane *phenomenology* [33][34]. In this discussion, we will discover also that the two approaches for compensating the RR charge of a half D6-brane (with a D8-brane or with the massive type IIA background) are not so unrelated as they may seem at a first look. The D8-brane is charged under the RR form $A^{(9)}$. The $A^{(9)}$ gauge potential is exceptional in type IIA string, since it has no dual potential. Its field strength does have a dual field strength but the field equations restrict this to be a constant, m . This constant is essentially the square root of the cosmological constant appearing in the massive IIA supergravity theory [32]. The equation of motion for $A^{(9)}$ in the type IIA string in the presence of D8-branes (which we take to be a point, say, in the x^7 direction) implies that $F^{(10)}$ is piece-wise constant function away from the D8-branes with a jump of one unit when we cross one D8-brane [33]. This results in a step-like non-zero cosmological constant, proportional to the square of the constant value of $*F^{(10)}$. In the region of space between two different D8-branes the bulk fields are conveniently described by the massive type IIA supergravity. In the units that we use in this paper, the parameter m is an integer which jumps by n units when one meets n coincident D8-branes⁶.

The D8-branes, whose introduction we tried to avoid, are apparently coming back in the game, since the only known supersymmetric backgrounds that solve the IIA field equations for $m \neq 0$ require the presence of eight-branes. However, we may assume that the D8-branes are positioned far away at infinity in the direction x^7 and that their only role is to provide a non-zero cosmological constant.

⁶ The appropriate normalizations for fields and charges are discussed in the appendix, where the exact relation between m and the number of D8-branes and their charge is derived. In this paper, all the RR charges are set to 1, unless explicitly said.

In this set-up, half D6-branes can exist provided that they end on parallel NS-branes and the deficit of RR charge is compensated by a suitable value of the cosmological constant. Let us restate the condition which follows from (2.10) reintroducing the charge μ for the D6 and the NS branes, for the configuration in which n_6 D6-branes end on n_{ns} NS-branes,

$$n_6\mu_6 = mn_{ns}\mu_{ns}. \quad (2.11)$$

The parameter m is equal to the μ_8 charge of the D8-branes times the number of such D8-branes. It is an highly non-trivial check that the actual value of the various charges allows a solution in terms of three integers, the number of D6, NS and D8 branes. The condition becomes,

$$n_6 = mn_{ns} \quad (2.12)$$

where m is now normalized, up to an integer factor, to correspond to the number of D8-branes present at infinity. The check of value of the charges will be performed in the appendix.

As we saw, the two approaches for reabsorbing the RR flux of an half D6-brane are closely related. We can always create a cosmological constant simply by moving a decoupled D8-brane sitting at infinity to the other side of our system. According to the previous discussion, a semi-infinite D6-brane can now end on the NS' brane. To be more precise, a finite D6-brane *must* be now stretched between the NS'-brane and the D8-brane. Every time a D8-brane crosses a NS'-brane, a D6-brane is created between them. This is nothing but a “T-dual” version of the effect discovered in [2]. This will be the subject of section 3. To simplify the discussion we will consider the system consisting only of D6, D8 and NS'-branes, which realizes a six dimensional gauge theory.

3. More about D6, D8 and NS'-branes

3.1. Gauge Theory in six-dimensions

Before discussing the application of this mechanism to the understanding of the chiral symmetry in four dimensional SQCD realized as a configuration of D4-branes, we want to apply this construction to a simpler model describing six-dimensional gauge theories. We will obtain a consistency check for our approach by deriving the anomaly restrictions for an $SU(N)$ six-dimensional gauge theory, and, as a by-product, we will determine the actual relation between the parameter m and the number of D8-branes.

Thus, we learn that in order to understand some aspects of chiral gauge theories in four dimensions we need to study the dynamics of six dimensional theories. A “T-dual” version of this approach was considered in [4]. There, the study of five dimensional gauge theories which can be realized by branes in type IIB was performed. It was found that each such model gives rise to a three dimensional gauge theory by stretching a D3 brane between a pair of fivebrane polymers. We will use the same strategy here by studying first six dimensional theories which can be realized in terms of branes and then use such systems to the study of four dimensional theories using D4-branes.

Consider a system with NS', D6 and D8-branes. Their world-volume is as indicated in the dictionary in section 2.1. In the directions $(x^0, x^1, x^2, x^3, x^8, x^9)$ which are common to all the branes a $N = 1$ six-dimensional gauge theory is realized. In the set-up of section 2.1, this six-dimensional gauge theory must be reinterpreted as the background global symmetry of the gauge theory living on the D4-branes. For the moment, we will forget about D4-branes and we will consider the six-dimensional theory on its own. This configuration with NS', D6 and D8-branes can be considered as the T-dual of the system originally proposed in [2] and has been studied recently in [5].

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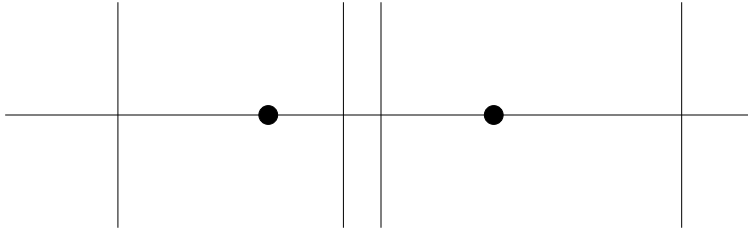


Figure 2: A system of D6 NS' and D8 branes. The D6 branes are stretched along the x^7 direction and are represented by horizontal lines, the NS' branes are localized in this direction and are represented by points and the D8 branes are localized in this direction and are represented by vertical lines.

We can picture, as in figure 2, a straight line, the x^7 direction, along which the D6-branes are stretched. The NS'-branes are points on the line on which the D6-branes are assumed to end. The D8-branes intersect the line and the 68-open strings give rise to hypermultiplets on the world-volume of the D6-branes.

Consider the simplest example with only two NS'-branes [5] and N D6-branes along the x^7 direction. We can now break the D6-branes between the NS', considering the theory

on the semi-infinite D6-branes on the left or on the right of the NS'-branes as a frozen theory with zero coupling constant. From the field theory point of view, we are dealing with an $SU(N)$ gauge theory on the world-volume of the N D6-branes stretched between the two NS'-branes, with $2N$ hypermultiplets coming from the open strings which connect the D6-branes with the semi-infinite D6-branes on the left or on the right.

The matter content of a $N = 1$ six-dimensional gauge theory is highly constrained by the gauge anomaly [35][36]. If the gauge group has an independent fourth order Casimir element (and this is the case for $SU(N)$, at least for $N > 3$), the gauge anomaly cancellation constrains the matter content. For $SU(N)$, the number of flavors must be twice the number of colors. As a general rule, it is believed that the charge conservation for the bulk fields corresponds to the anomaly cancellation on the world volume of the branes. In our example, the conservation of the RR charge for the D6-branes corresponds to the statement that the number of flavors is $2N$, which is exactly the condition for the anomaly cancellation in field theory. It should be noted that even for number of flavors equal to twice the number of colors, the anomaly must be cancelled by the introduction of a tensor field [35]. Here, a tensor field in the six-dimensional theory is automatically provided by each of the NS'-branes [5]. One of the two tensors parametrizes the center of mass motion of the system and it is decoupled from the theory. The scalar ϕ , partner of the dynamical tensor under $N = 1$ supersymmetry, is associated with the distance between the two NS'-branes. The same distance is also the gauge coupling of the theory. The fact that the two quantities coincide corresponds to the well-known fact [35] that in the Lagrangian

$$\frac{1}{g^2} F_{\mu\nu}^2 + (\partial\Phi)^2 + \sqrt{c}\Phi F_{\mu\nu}^2 \quad (3.1)$$

$\frac{1}{g^2}$ can be absorbed in ϕ .

We can easily generalize this construction by introducing, as in figure 3, n D8-branes which intersect the x^7 line in points between the two NS'-branes and considering now a different number n_l (n_r) of semi-infinite D6-branes on the left (right) of the NS'-branes. The mismatch of charge between the left and the right of each NS'-brane can be compensated by the cosmological constant induced by the D8-branes. We will assume that there are other D8-branes at infinity in the x^7 direction in such a way that the value of the parameter m on the left of the first NS'-brane is p . Every time we meet a D8-brane on the x^7 axis the value of m jumps by 1 unit. This means, for example, that the value of m on the right of the second NS'-brane is $p + n$.

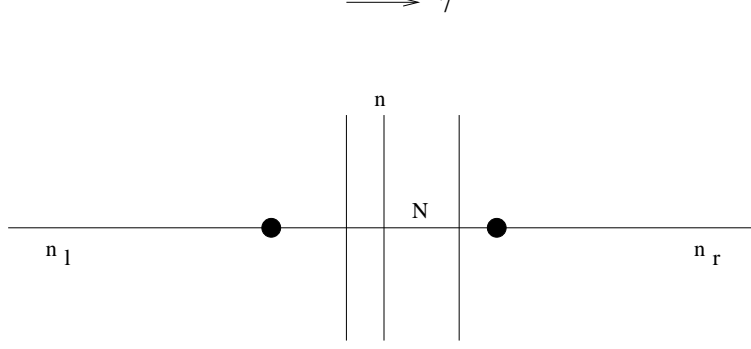


Figure 3: Six dimensional theory realized in terms of NS', D6 and D8 branes. There are N D6 branes stretched in between two NS' branes. $n_l(n_r)$ D6 branes end to the left (right) of the NS' branes. In addition there are n D8 branes in between the two NS' branes. The configuration describes $SU(N)$ gauge theory with $n + n_l + n_r$ flavors coupled to one tensor multiplet.

We expect that the charge conservation still gives the anomaly cancellation constraint. The two charge conservation equations now read:

$$\begin{aligned} N - n_l + p &= 0 \\ n_r - N + p + n &= 0 \end{aligned} \tag{3.2}$$

and we indeed get the relation that the total number of flavors $N_f = n_l + n_r + n$ must be equal to $2N$. Note that the enhanced non-Abelian flavor symmetry in this case is $SU(n_l) \times SU(n) \times SU(n_r)$ for the case in which all three families of branes meet. This is a subgroup of the expected flavor symmetry which is $SU(N_f)$. The full $SU(N_f)$ symmetry is visible, for example, if all flavors are given by D8 branes in between the two NS' branes. That is for $n_l = n_r = 0$ and $n = 2N$ when the x^7 positions of the D8 branes coincide we see a $SU(N_f)$ symmetry.

3.2. Creation of a sixbrane

The exact value of the x^7 position of the D8-branes did not enter in the previous discussion. This is obviously related to the fact that hypermultiplets can not be massive in six-dimensions, so we do not expect parameter associated with them. The role of their position is analogous to the x^6 position of the D5-branes in [2]; this parameter is not visible as a deformation of the quantum field theory, but it may be associated to non-trivial phase transition corresponding to moving a D8-brane on the other side of a NS'-brane. Exactly as in [2], where a D3-brane appears when a D5-brane passes through a NS brane, we expect

that D6-branes can be created when a D8-brane passes through a NS'-brane. The charge conservation tells us whether this happens or not.

Before studying some six dimensional properties related to the motion of the D8 branes let us mention a fact which will be useful later. We can always think that the semi-infinite D6-branes end on D8-branes located at infinity. This can be done without loss of generality due to the following process. Suppose we have an infinite D6 brane and let us put a D8 brane in some very large value of x^7 . The two branes touch and thus the D6 brane can break on the D8 brane. One side is stretched to the origin of x^7 and the other is stretched far at large x^7 values. Now we can move this latter semi-infinite D6 brane to large values. Thus we are left with a D6 brane which ends on a D8 brane at very far values of x^7 . The gauge theory which is located around the origin of x^7 is not affected from this process.

Let us consider the simplest example, $n_l = n_r = N$ and no D8-branes $n = 0$ (a configuration consistent with a zero cosmological constant). This is a $U(N)$ gauge theory with $2N$ hypermultiplets. We use what we have said in the last paragraph and assume that the semi-infinite D6-branes end on D8-branes located at infinity. Now move one of these D8-branes located at $x_7 = -\infty$ to the right of the first NS'-brane. One semi-infinite D6-brane disappears and one D8-brane is now in between the two NS'-branes. The theory is still a $U(N)$ gauge theory with $2N$ hypermultiplets. No new D6-brane is appeared. The relations $n_l = N - 1, n = 1, n_r = N$ are consistent since the apparent lack of charge at the position of the first NS'-brane is compensated by the value $m = -1$ created by the D8-brane on the region of space on its left. But when the D8-brane moves to the right of the second NS'-brane a D6-brane is created between the NS' and the D8-brane. $n_l = N - 1, n = 0, n_r = N + 1$ is indeed the only configuration which satisfies the charge conservation. This follows from the fact that the cosmological constant has been turned on ($m = -1$) in the whole region which contains the two NS'-branes. We can now pull the D8-brane to the positive infinity and we remain with the same gauge theory, $U(N)$ with $2N$ hypermultiplets, but with a different arrangement of semi-infinite branes on the left and on the right, $n_l = N - 1, n_r = N + 1$.

Since the configuration of NS, D8 and D6-branes is T-dual to the system considered in [2], we expect that the creation of D6-branes is also regulated by a linking number argument. We want now to show that the RR charge conservation and the linking number argument are actually equivalent.

Let us briefly review the linking number argument. The presence of a NS' brane generates a non-trivial background for the B field. This affects the D8 world-volume theory since the D8 $U(1)$ gauge field mixes with the bulk field B in such a way that $F_{D8} - B$ is the gauge invariant curvature. We start with a general configuration of D6-branes ending on D8-branes. The endpoints of the D6-branes look like monopoles with centers in (x_i^4, x_i^5, x_i^6) for the D8 world-volume gauge field,

$$d(F_{D8} - B) = dF_{D8} - H = \sum_i \delta_{D6}(x_i^4, x_i^5, x_i^6) \quad (3.3)$$

If we integrate this equation over the R^3 spanned by (x^4, x^5, x^6) , $\int dF_{D8}$ gives the total magnetic charge for the $U(1)$ gauge field. This quantity is a constant and because it can be measured at infinity it is unaffected by the motion of the branes in spacetime. We conclude that the sum of $\int_{D8} dx^4 dx^5 dx^6 H$ on the world-volume of the D8-brane with the total net number of D6-branes ending on the D8 must be conserved. $\int_{D8} dx^4 dx^5 dx^6 H$ still depends on the position in x^7 of the D8-brane and it is easily checked that it has a different sign whether the D8-brane is on the left or on the right of the NS' brane. The jump of 1 unit⁷ which occurs when the D8-brane passes through the NS'-brane is compensated by the creation of a D6-brane. This is the phenomenon that we explained before using the RR charge conservation for the type IIA 1-form. This is not a mere coincidence occurring in this specific example and the two arguments in six dimensions are equivalent. This means that we do not have to double the number of constraints which regulates the allowed configurations and the phase transitions, but simply use the one that we prefer. The equivalence can be explicitly checked for a generic configuration of D6, D8 and NS'-branes, once one remembers that, for the S-rule, no more than one D6-brane can end on a D8-brane if the other end lives on a NS'-brane. It is easy to see that the linking number and the charge conservation argument coincide, provided $m(x^7)$ is assumed to jump by 1 unit at the location of each D8-brane. This assumption was used constantly in this section and will be proved in the appendix. The previous discussion can also be considered as a further check of this assumption.

⁷ The actual value of the jump was derived in [2] using the geometrical interpretation of the constant magnetic charge as the linking number invariant when the D5 and NS-worldvolume is conveniently compactified. The value follows also from the equations of motion when normalizations for fields and charges are carefully taken into account. This will be done explicitly in the appendix. As already noted, there is a non-trivial cancellation of RR charge quanta which results in an integer.

4. Chiral symmetry in four dimensions

4.1. Localized chiral matter

We now return to SQCD in four-dimension. We will show that using the mechanism described in the previous sections, we can make the chiral symmetry manifest on the brane configuration by localizing matter of different chirality in different points.

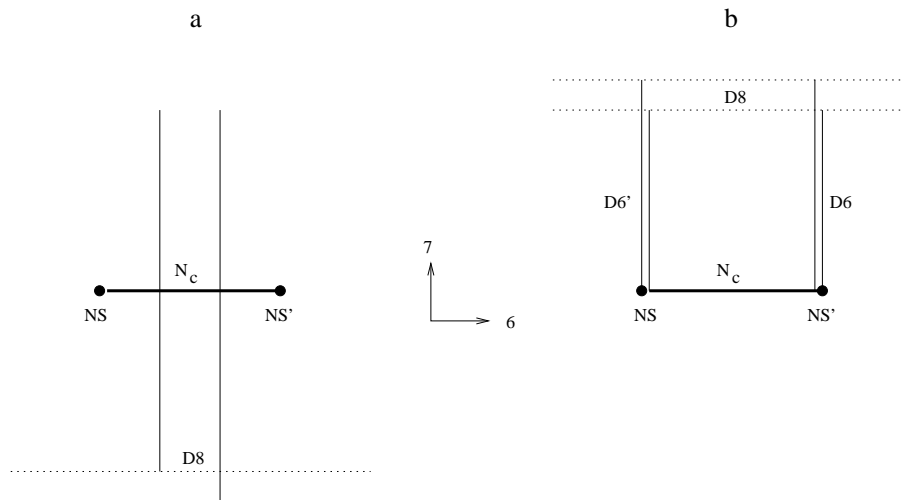


Figure 4: Localization of chiral matter in $N = 1$ supersymmetric QCD. In figure (a) the usual brane configuration which describes this theory is sketched. Vertical lines represent D6 branes which end on D8 branes that are located very far in the x^7 direction. Hypermultiplets arise at points where sixbranes and fourbranes meet. Figure (b) describes the resulting configuration after the D8 branes move to the other side of the x^7 coordinate. Chiral multiplets are localized at the two ends of the fourbranes, on the fivebranes world volume.

$SU(N_c)$ SQCD is realized by stretching N_c D4-branes in the x^6 direction between a NS and a NS'-brane. When the N_f D6-branes are tuned to touch the NS'-brane, each of them splits into two semi-infinite D6-branes. Each of the D6-halfbranes supports a $SU(N_f)$ gauge group and we can see the full $SU(N_f)_l \times SU(N_f)_r$ chiral symmetry of the four dimensional gauge theory. We want to give further evidence for this proposal [13] by using the construction of the previous sections.

We can imagine, without loss of generality, that the D6-halfbranes end on D8-branes located at infinity in the x^7 direction. This can be done by the following process which has the interpretation of a Higgs mechanism for the six dimensional theory. We can take N_f D8 branes and break each of the D6 branes. Now there are two half six branes per

each D8 brane. One half is connected to a NS' brane far away and by the rules of section 2.1 and [2] can not move. The other half is free to move along the 456 directions and can be taken far away in these directions. Note that we need N_f D8 branes in order to avoid S-configurations which are not supersymmetric [2]. The resulting configuration is depicted in figure 4(a). The cosmological constant can be tuned (by adding other far away D8 branes) to be zero in the region of spacetime which contains our brane system. Now move the D8-branes which are located at negative infinity in x^7 to positive infinity in x^7 . They have to pass through the NS and NS'-branes and thus D6-branes can be created according to the discussion in the previous section. The only difference is that if a Dirichlet 6-brane is created with an end on the NS-brane, it must be parallel to it, so it must be what we called a $D6'$ -brane. The value of the cosmological constant is now $m = N_f$. It is easy to enforce the charge conservation. In the anomaly free final configuration, shown in figure 4(b), there are N_f D6-halfbranes ending on the NS'-brane and N_f $D6'$ -halfbranes ending on the NS-brane. Both of them are along the positive x^7 axis. The apparent lack of RR charge at both the NS and NS' location is compensated by the non-zero value of the cosmological constant.

The gauge theory on the world-volume of the D4-brane is still $SU(N_c)$ with N_f fields Q in the fundamental and N_f fields \tilde{Q} in the anti-fundamental, but now Q is supported near the NS end of the D4-brane, while \tilde{Q} is supported near the NS' end. The global symmetry is manifestly $SU(N_f)_l \times SU(N_f)_r$, and the two chiral factors are now supported by D6-halfbranes at different positions. This gives evidence to the proposal in [13] for manifestly realizing the chiral symmetry by splitting the D6-branes.

Charge conservation in spacetime should correspond to anomaly cancellation on the branes. The final gauge theory is manifestly anomaly free, but the anomaly cancellation is more subtle and it is worthwhile to comment on it. The world-volume quantum field theory on the D4-brane is actually a five-dimensional gauge theory defined on R^4 times a segment. A five-dimensional theory can not have anomaly. However the two boundaries are four-dimensional and the boundary values of the spinors can give anomalies. In the KK reduction all the massive fields do not contribute to the anomaly, and the global anomaly is given by the massless fields. The theory for these massless fields is $SU(N_c)$ with N_f fields Q in the fundamental and N_f fields \tilde{Q} in the anti-fundamental for both the cases $m = 0$ and $m \neq 0$ and it is anomaly free. However the anomaly must cancel also locally on each boundary. In the $m \neq 0$ case each boundary supports only the fields Q or the fields \tilde{Q} , so the anomaly is apparently not cancelled. However, as noted in [37], in the presence

of a cosmological constant there are extra terms on the world-volume of a D-brane. In particular, on a D4-brane there is a Chern-Simons term

$$m \int dx^5 \omega_5 \quad (4.1)$$

where ω_5 is the Chern-Simons 5-form with the property that $d\omega_5 = \text{tr} F^3$. Under a gauge transformation, $\delta\omega_5 = d(F \wedge F)$. We see that such a term is not gauge invariant when the theory is defined on a segment, and gives an anomaly supported on the boundary,

$$\delta S_{D4} = m \int dx^5 d(F \wedge F) = m \int dx^4 (F \wedge F)|_1 - m \int dx^4 (F \wedge F)|_2 \quad (4.2)$$

where 1 and 2 refers to the two boundaries. We see that this anomaly has exactly the sign and the magnitude necessary to cancel the anomaly of the fields Q and \tilde{Q} supported on the two boundaries⁸.

This contribution from the Chern-Simons terms is always present when the cosmological constant is non-zero. For what regards anomalies, it is harmless for infinite branes. But when the D-brane is defined on a segment, it produces anomalies localized at the boundaries. Starting with a D4-brane stretched between NS-branes in the presence of a cosmological constant and nothing else, we would find an anomalous theory and we would start looking for chiral matter localized at the boundaries, discovering the necessity of half D6-branes. This anomaly argument provides further support for the proposal [13] of how chiral matter arises.

The final configuration with non-zero cosmological constant indicates clearly the origin of the chiral symmetry on the D6 halfbranes. The fields of different chirality are now supported on different halfbranes and it is tempting to speculate that with this mechanism we could construct chiral models and even realize a smooth process that connect non-chiral and chiral gauge theories in a Seiberg's duality. Example of this kind do actually exist in the literature.

⁸ The anomaly in (4.2) clearly cancels in the KK analysis and does not contribute to the global anomaly.

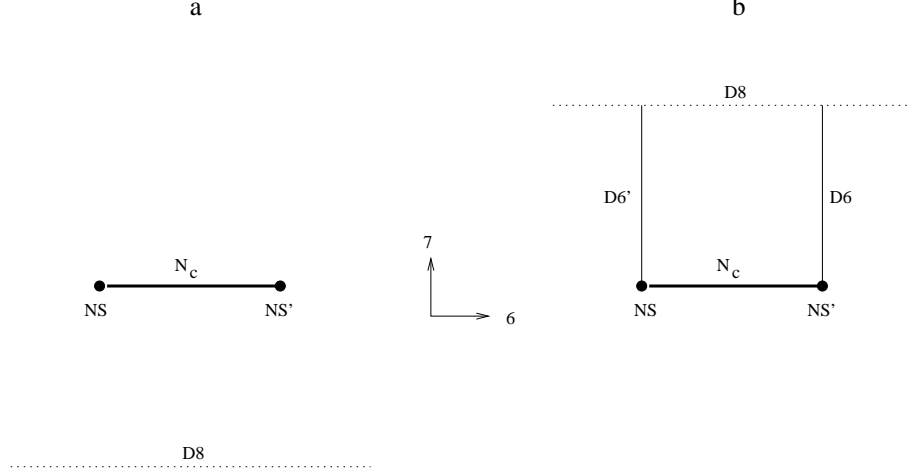


Figure 5: A phase transition in which the number of flavors for a gauge theory are affected by the ten dimensional cosmological constant. In figure (a) there are N_c fourbranes between a NS brane and a NS' brane. A D8 brane is in the large negative x^7 position. In figure (b) the D8 brane moves to the far positive x^7 position and creates two D6 branes which connect to the two NS branes. A D6 is created from passing the NS' brane and a D6' is created when passing the NS brane.

4.2. A curious phase transition

In the SQCD example of the previous section, the original and final quantum field theories were actually the same. It is easy to produce an example in which, by moving a D8-brane, we can connect two theories with different field content.

Consider N_c D4 branes connecting a NS branes to a NS'-brane. Let the two NS branes be positioned at the origin of the x^7 direction. This configuration describes $N = 1$ supersymmetric $SU(N_c)$ gauge theory with no matter. Consider, as in figure 5(a), a D8 brane positioned at a very far negative $x^7 = m_0$ position. Such a D8 brane causes a jump in the value of the cosmological constant.

$$\begin{aligned} m(x^7) &= -1 & x^7 < m_0 \\ m(x^7) &= 0 & x^7 > m_0. \end{aligned} \tag{4.3}$$

The value of the cosmological constant is zero at the origin in x^7 and so the four dimensional system is not affected by the presence of the D8 brane far away. Now move one D8-brane, as in figure 5(b), from the large negative position to a positive large position in the x^7 direction. The cosmological constant at the origin of the x^7 direction, where the NS branes are located, is now $m = -1$. Our discussion from section 3.2 implies that a D6 and D6'-branes are created, ending on the NS'-brane and the NS-brane, respectively. The apparent

paradox that there is no charge conservation on the NS branes where the D6 branes end, is now avoided by the fact that the cosmological constant has changed its value, as explained in section 2.5.

We now use our proposed dictionary for matter content as in section 2.1 and find [13] that the matter content is now of a field Q in the fundamental representation and a field \tilde{Q} in the anti-fundamental representation of the gauge group. The field Q comes from strings stretching between the D6 brane and the D4 branes while the field \tilde{Q} comes from strings stretching between the D6' brane and the D4 branes. The theory is now SQCD with one quark flavor.

We saw that we can change the number of flavors of the SQCD theory simply by moving D8-branes on the other side of the system. This corresponds to a change in the value of the cosmological constant in the region where the D4-branes live. The number of flavor depends on the cosmological constant, which is indeed quantized. The curious thing is that we have changed the number of flavors by changing a real parameter in spacetime - the position of the D8-brane. This phenomenon has no known explanation in terms of the field theory living on the D4-brane, since the only way to make the quarks massive in a $N = 1$ theory involves complex parameters.

5. Conclusions

We have shown how to localize chiral matter in different spacetime points in a system of D-branes in the type IIA string which gives a realization of $N = 1$ gauge theories in four dimensions.

The ultimate goal would be to realize chiral models and, eventually, study Seiberg's dualities associated. There exists in the literature examples of chiral models dual to non-chiral gauge theories [38]. It would not be surprising that the phase transitions considered in this paper can be generalized to cover these cases, too. Charge conservation simply says that if we start with an anomaly free theory, we must end after some phase transition with another anomaly free theory. But it does not exclude the option to end up with a chiral theory, if we started with a non-chiral one.

The main problem to solve for realizing chiral models is how to construct theories with chiral two-indices tensors. In this paper, we have shown how to get rid of chiral fields in the fundamental of the gauge group. $SU(N)$ theories with chiral fundamental are anomalous (and indeed we did not get them by phase transition in this paper) unless we

compensate the anomaly with appropriate chiral two-indices tensors. These tensors do not arise naturally in the brane set-up of this paper. Some new ingredient is required. This issue is currently under investigation.

Appendix A. Normalization of RR charges

In this appendix we want to discuss the normalization for the RR and NS brane charges appearing in (2.11) and the relation between m and the number of D8-branes.

We will use the following normalizations for H and $F^{(2)}$ in the massive type IIA Lagrangian,

$$S_{IIA} = -\frac{1}{(2\pi)^7 \alpha'^4} \int d^{10}x \left(e^{-2\phi} \frac{1}{2} H \wedge *H \right) + \frac{1}{2} (F^{(2)} - mB) \wedge *(F^{(2)} - mB). \quad (\text{A.1})$$

With this normalization for H , the coupling to the fundamental string is $(2\pi\alpha')^{-1} \int B$. The coupling between Dp-branes and RR fields is as follows,

$$S = \frac{1}{2} \int d^{10}x F^{(p+2)} \wedge *F^{(p+2)} + i\mu_p \int d^{p+1} A^{(p+1)}. \quad (\text{A.2})$$

The value of the RR charge was found in [1] and reads,

$$\mu_p^2 = 2\pi(4\pi^2\alpha')^{3-p}. \quad (\text{A.3})$$

They satisfy the Dirac quantization condition,

$$\mu_p \mu_{6-p} = 2\pi. \quad (\text{A.4})$$

It was suggested in [39] that the cosmological constant in the massive type IIA theory is quantized. This is consistent with the fact that the only known supersymmetric solutions of the equations of motion require the existence of D8-branes and that the parameter m is proportional to the number of D8-branes. A simple argument [39] allows to determine the exact proportionality coefficient. Consider the field equation

$$\frac{1}{(2\pi)^7 \alpha'^4} d * (e^{-2\phi} H) = m * (F^{(2)} - mB) \quad (\text{A.5})$$

and integrate it over an 8-sphere surrounding a D0-brane. The inconsistent result that the flux $\int *F^{(2)}$ is zero when $m \neq 0$ is modified if we suppose that a fundamental string ends on

the D0-brane and intersects the 8-sphere (this is exactly what a D-brane is: a manifold on which the open strings end⁹). Such a string contributes to the equation of motion of B with a delta-function times $(2\pi\alpha')^{-1}$. The condition is now $m \int *(F^{(2)} - mB) - (2\pi\alpha')^{-1} = 0$ or

$$m\mu_0 = \frac{1}{2\pi\alpha'} \quad (\text{A.6})$$

Using formula (A.3), we learn that $m = \mu_8$. This means that the presence of one D8-brane changes by 1 unit the value of m , a fact that we used constantly in the paper.

The last thing we need to show is that in formula (2.11) the quanta of RR and NS charge exactly cancel, giving the relation between the number of D6, NS and D8 branes in formula (2.12). With the conventions of this appendix, the relevant equation is

$$n_8\mu_8 dH = n_6\mu_6\delta^{(4)} \quad (\text{A.7})$$

Using the quantization condition (A.6) and the relation (A.4), this is equivalent to

$$\frac{n_8 dH}{2\pi\alpha'} = n_6\mu_o\mu_6\delta^{(4)} = 2\pi n_6\delta^{(4)}. \quad (\text{A.8})$$

The coupling of a NS-brane can be explicitly derived by a dimensional reduction of the M-theory five-brane [40]. Consider the relevant part of the eleven dimensional supergravity Lagrangian,

$$-\frac{1}{(2\pi)^8\alpha'^{\frac{9}{2}}} \int \left(\sqrt{-G}R + \frac{1}{2}F_M^{(4)} \wedge *F_M^{(4)} \right). \quad (\text{A.9})$$

The KK reduction on a circle gives the type IIA Lagrangian if we use the ansatz [41],

$$ds^2 = e^{-\frac{2}{3}\phi} G_{\mu\nu} dx^\mu dx^\nu + e^{\frac{4}{3}\phi} (dy - A_\mu^{(1)} dx^\mu)^2, \quad A_M^{(3)} = A^{(3)} + B^{(2)} \wedge dy. \quad (\text{A.10})$$

where y is the eleventh dimension coordinate with periodicity $2\pi\sqrt{\alpha'}$. The normalization for H in the reduced Lagrangian is as in (A.1), while the RR fields need to be rescaled by a factor $(2\pi)^{7/2}\alpha'^2$.

⁹ More precisely, it was pointed out in [25] that whenever there is a cosmological constant a fundamental string *must* be attached to the D0-brane. In general, when a D8-brane crosses a D0-brane (creating a non-zero cosmological constant in the region in which the D0-brane lives) a fundamental string is created in between them. This phenomenon is U-dual to the creation of a D6-brane in between D8 and NS'-branes considered in this paper.

The five-brane charge in M-theory

$$\mu_{5M} \int A_M^{(6)} \quad (\text{A.11})$$

can be easily determined by identifying the M five-brane wrapped around the eleventh dimension with the D4-brane of type IIA¹⁰

$$2\pi\sqrt{\alpha'}(2\pi)^{7/2}\alpha'^2\mu_{5M} = \mu_4 \quad (\text{A.12})$$

The reduced (not wrapped) M five-brane must be identified instead with the type IIA NS-brane. Reducing the coupling (A.11) we get¹¹

$$\frac{e^{-2\phi}}{(2\pi)^5\alpha'^3} \int *B^{(6)}. \quad (\text{A.13})$$

The equation for the NS-brane is now,

$$dH = n_{ns}(2\pi)^2\alpha' \quad (\text{A.14})$$

When substituted in (A.8), it gives the desired relation $n_8 n_{ns} = n_6$. The quanta of R and NS charge have disappeared. As noted in the text, this is an highly non-trivial result.

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¹⁰ Remember that the RR fields must be rescaled to agree with our notations.

¹¹ Note that the definition of the Poincare dual involves the metric. The ten dimensional metric is obtained from the eleven dimensional one by a Weyl rescaling. This produces the extra dependence on the dilaton.

References

- [1] J. Polchinski, *Dirichlet-Branes and Ramond-Ramond Charges*, Phys. Rev. Lett. 75 (1995) 4724
- [2] A. Hanany and E. Witten, *Type IIB Superstrings, BPS Monopoles, And Three-Dimensional Gauge Dynamics*, IASSNS-HEP-96/121, hep-th/9611230.
- [3] E. Witten, *Solutions Of Four-Dimensional Field Theories Via M Theory*, hep-th/9703166.
- [4] O. Aharony and A. Hanany, *Branes, Superpotentials and Superconformal Fixed Points*, hep-th/9704170.
- [5] I. Brunner and A. Karch, *Branes and Six Dimensional Fixed Points*, hep-th/9705022.
- [6] B. Kol, *5d Field Theories and M Theory*, hep-th/9705031.
- [7] J. de Boer, K. Hori, H. Ooguri, Y. Oz, Z. Yin, *Mirror Symmetry in Three-Dimensional Gauge Theories, $SL(2, \mathbb{Z})$ and D-Brane Moduli Spaces*, hep-th/9612131.
- [8] K. Landsteiner, E. Lopez, D. A. Lowe, *$N=2$ Supersymmetric Gauge Theories, Branes and Orientifolds*, hep-th/9705199.
- [9] A. Brandhuber, J. Sonnenschein, S. Theisen, S. Yankielowicz, *M Theory And Seiberg-Witten Curves: Orthogonal and Symplectic Groups*, hep-th/9705232.
- [10] S. Elitzur, A. Giveon, and D. Kutasov, *Branes and $N = 1$ Duality in String Theory*, hep-th/9702014.
- [11] J. de Boer, K. Hori, Y. Oz, Z. Yin, *Branes and Mirror Symmetry in $N=2$ Supersymmetric Gauge Theories in Three Dimensions*, hep-th/9702154.
- [12] N. Evans, C. V. Johnson and A. D. Shapere, *Orientifolds, Branes, and Duality of 4D Gauge Theories*, hep-th/9703210.
- [13] J.H. Brodie and A. Hanany, *Type IIA Superstrings, Chiral Symmetry, and $N = 1$ 4D Gauge Theory Duality*, hep-th/9704043
- [14] A. Brandhuber, J. Sonnenschein, S. Theisen, S. Yankielowicz, *Brane Configurations and 4D Field Theory Dualities*, hep-th/9704044.
- [15] S. Elitzur, A. Giveon, D. Kutasov, E. Rabinovici and A. Schwimmer, *Brane Dynamics and $N=1$ Supersymmetric Gauge Theory*, hep-th/9704104.
- [16] R. Tatar, *Dualities in 4D Theories with Product Gauge Groups from Brane Configurations*, hep-th/9704198.
- [17] S. Katz, C. Vafa *Geometric Engineering of $N=1$ Quantum Field Theories*, hep-th/9611090.
- [18] M. Bershadsky, A. Johansen, T. Pantev, V. Sadov, C. Vafa *F-theory, Geometric Engineering and $N=1$ Dualities*, hep-th/9612052.
- [19] C. Vafa, B. Zwiebach, *$N=1$ Dualities of SO and USp Gauge Theories and T-Duality of String Theory*, hep-th/9701015.
- [20] H. Ooguri, C. Vafa *Geometry of $N=1$ Dualities in Four Dimensions* hep-th/9702180.

- [21] C. Ahn, K. Oh *Geometry, D-Branes and N=1 Duality in Four Dimensions I*, hep-th/9704061.
- [22] C. Ahn *Geometry, D-Branes and N=1 Duality in Four Dimensions II*, hep-th/9705004.
- [23] C. Ahn, R. Tatar, *Geometry, D-branes and N=1 Duality in Four Dimensions with Product Gauge Group*, hep-th/9705106.
- [24] M. Berkooz, M. R. Douglas, R. G. Leigh, *Branes Intersecting at Angles*, hep-th/9606139, Nucl.Phys. B480 (1996) 265-278.
- [25] U. H. Danielsson, G. Ferretti and I. R. Klebanov, *Creation of Fundamental Strings by Crossing D-branes*, hep-th/9705084; O. Bergman, M. R. Gaberdiel and G. Lifschytz, *Branes, Orientifolds and the Creation of Elementary Strings*, hep-th/9705130.
- [26] C. Bachas, M. R. Douglas, M. B. Green, *Anomalous Creation of Branes*, hep-th/9705074.
- [27] O. Aharony and A. Hanany, unpublished.
- [28] J. L. F. Barbon, *Rotated Branes and N = 1 Duality*, CERN-TH/97-38, hep-th/9703051.
- [29] E. G. Gimon and J. Polchinski, *Consistency Conditions for Orientifolds and D-Manifolds*, Phys.Rev. D54 (1996) 1667, hep-th/9601038.
- [30] M. Green, J. A. Harvey and G. Moore, *I-Brane Inflow and Anomalous Couplings on D-Branes*, Class. Quant. Grav. 14 (1997) 47, hep-th/9605146.
- [31] M. R. Douglas, *Branes within Branes*, hep-th/9512077
- [32] L. Romans, *Massive N=2A supergravity in ten-dimensions*, Phys. Lett. B169 (1986) 374.
- [33] J. Polchinski and E. Witten, *Evidence for heterotic - type I string duality*, Nucl. Phys. **B460** (1996) 525.
- [34] J. Polchinski and A. Strominger, *New vacua for type IIA string theory*, Phys.Lett. B388 (1996) 736, hep-th/9510227.
- [35] N. Seiberg, hep-th/9609161; *Phys. Lett.* **B390** (1996) 753.
- [36] U. Danielsson, G. Ferretti, J. Kalkkinen and P. Stjernberg, hep-th/9703098.
- [37] M. B. Green, C. M. Hull, P. K. Townsend, *D-brane Wess-Zumino actions, T-duality and the cosmological constant* Phys. Lett. B382 (1996) 65, hep-th/9604119
- [38] P. Pouliot, *Chiral Duals of Non-Chiral SUSY Gauge Theories*, Phys.Lett. B359 (1995) 108, hep-th/9507018; P. Pouliot, M. J. Strassler, *A Chiral SU(N) Gauge Theory and its Non-Chiral Spin(8) Dual* Phys. Lett. B370 (1996) 76, hep-th/9510228; M. Berkooz, *The Dual of Supersymmetric SU(2k) with an Antisymmetric Tensor and Composite Dualities*, Nucl. Phys. B452 (1995) 513, hep-th/9505067; K. Intriligator, R. G. Leigh, M. J. Strassler, *New Examples of Duality in Chiral and Non-Chiral Supersymmetric Gauge Theories*, Nucl.Phys. B456 (1995) 567, hep-th/9506148.
- [39] J. Polchinski and A. Strominger, *New Vacua for Type II String Theory*, Phys.Lett. B388 (1996) 736.

- [40] S. P. de Alwis, *Coupling of branes and normalization of effective actions in string/M-theory*, hep-th/9705139
- [41] E. Witten, *String Theory Dynamics In Various Dimensions*, Nucl.Phys. B443 (1995) 85, hep-th/9503124.